

Nonlocal Nambu-Jona-Lasinio model and chiral chemical potential

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Abstract

We derive the critical temperature in a nonlocal Nambu-Jona-Lasinio model with the presence of a chiral chemical potential. The model we consider uses a form factor derived from recent studies of the gluon propagator in Yang-Mills theory and has the property to fit in excellent way the form factor arising from the instanton liquid picture for the vacuum of the theory. Nambu-Jona-Lasinio model is derived from quantum chromodynamics providing all the constants of the theory without any need for fits. We show that the critical temperature in this case always exists and increases as the square of the chiral chemical potential. The expression we obtain for the critical temperature depends on the mass gap that naturally arises from Yang-Mills theory at low-energy as also confirmed by lattice computations.

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I. INTRODUCTION

Recent studies on lattice show that Yang-Mills theories develop a mass gap in the low energy limit. This is seen both in the spectrum [1, 2] and for the gluon propagator [3–5]. On the theoretical side, several proposals have been put forward [6–12] but none of them reached the status of a rigorous proof. Notwithstanding this difficulty, this fundamental result can be used to understand quantum chromodynamics (QCD) in the infrared limit. A very good approximation for the gluon propagator in the Landau gauge at lower energies is a free massive propagator as can be deduced from aforementioned references.

Existence of a mass gap and an analytical equation for the gluon propagator in a given gauge opens up the possibility to perform computations at low energies in QCD both at zero and finite temperature. We were able to prove in this way that a nonlocal Nambu-Jona-Lasinio (nNJL) model describes the low energy phenomenology of hadron physics [13–17]. In Ref.[17] we obtained the critical temperature at zero chemical potential for the chiral transition. This turns out in close agreement with lattice data [18] and with preceding theoretical computations [19]. nNJL model was extensively studied in Ref.[20] that we will follow in this paper.

In this paper we aim to analyze a problem arisen when unbalanced chiral quark matter is present in a quark condensate. The question is how critical temperature changes due to the presence of chiral matter. This question was faced in [21] to identify the critical end point of QCD. The idea is, at the thermodynamic equilibrium, to couple the chiral chemical potential, μ_5 , to a chiral density quark operator, as also happens for the quark number density $\bar{\psi}\gamma_0\psi$ to the conjugated quark chemical potential μ [21–31] and references therein. Quantum anomaly and chirality changing processes makes μ_5 coupled to a not strictly conserved quantity. So, observation is assumed to take longer times than any chirality changing process. Recent theoretical studies support the idea that the critical temperature should decrease with chiral chemical potential [21–26]. Recent lattice data have shown that critical temperature increases with μ_5 [27, 28]. This behavior of $T_c(\mu_5)$ was predicted for the first time by universality arguments in [30] and it has also been found later by solving Schwinger-Dyson equations at finite μ_5 [31]. Recently, Ruggieri and Peng [32] draw this conclusion with a quark-meson model. We will strongly support their conclusions.

With our approach, we will show that a nNJL model provides a critical temperature

increasing with the chiral chemical potential. We will show that, with the results provided in literature for the gluon propagator, the mass gap equation obtain always a solution both with $\mu_5 = 0$ and $\mu_5 \neq 0$ and, in the latter case, it will depend on the square of it increasing the temperature. The mass gap seems to play a role, mostly as a lower threshold.

The paper is so structured. In Sec. II we discuss the infrared limit for QCD obtaining the model we solve in the low-energy limit. In Sec. III we derive the critical temperature. Finally, in Sec. IV conclusions are given.

II. INFRARED LIMIT

For the sake of completeness, we yields here a rederivation of the nonlocal Nambu-Jona-Lasinio model we derived in our preceeding works. Details can be found in [13–17].

In order to present our arguments, we consider the simplest nonlinear theory: a scalar field with equation

$$\square\phi + \lambda\phi^3 = j. \quad (1)$$

This equation has exact solutions [33] $\phi_0(x) = \mu (2/\lambda)^{\frac{1}{4}} \text{sn}(p \cdot x + \theta, i)$ being sn an elliptic Jacobi function and μ and θ two integration constants. This solutions hold provided the dispersion relation $p^2 = \mu^2 \sqrt{\lambda/2}$ applies. This is a free massive solution notwithstanding we started from a massless theory. Mass arises from the nonlinearities provided λ stays finite rather than going to zero. Indeed, standard perturbation theory just fails to recover it. We have to solve the equation (1) in the limit $\lambda \rightarrow \infty$. For our aims we consider an approach devised in the '80s [34]: we assume ϕ as a functional of j and take a power expansion of it. We put

$$\phi[j] = \phi_0(x) + \int d^4x' \left. \frac{\delta\phi[j]}{\delta j(x')} \right|_{j=0} j(x') + \int d^4x' d^4x'' \left. \frac{\delta^2\phi[j]}{\delta j(x')\delta j(x'')} \right|_{j=0} j(x')j(x'') + \dots \quad (2)$$

being $\left. \frac{\delta\phi[j]}{\delta j(x')} \right|_{j=0} = \Delta(x - x')$ a solution to the nonlinear equation $\square\Delta(x) + 3\lambda[\phi_0(x)]^2\Delta(x) = \delta^4(x)$. A full theory can be devised with this approach and an exactg propagator can be obtained [35]. This analysis shows that the theory is trivial. The propagator is given by [33]

$$\Delta(p) = \sum_{n=0}^{\infty} \frac{B_n}{p^2 - m_n^2 + i\epsilon} \quad (3)$$

being

$$B_n = (2n+1)^2 \frac{\pi^3}{4K^3(i)} \frac{e^{-(n+\frac{1}{2})\pi}}{1 + e^{-(2n+1)\pi}} \quad (4)$$

and $m_n = (2n + 1)(\pi/2K(i))(\lambda/2)^{\frac{1}{4}}\mu$ and $K(i) \approx 1.3111028777$ an elliptic integral. This holds provided one fixes the phase θ in the exact solution to $\theta_m = (4m + 1)K(i)$ to preserve translation invariance in the propagating degrees of freedom. This identifies an infinite set of scalar field theories with a trivial infrared fixed point in quantum field theory.

This can be immediately applied to Yang-Mills theories as we have exact solutions also in this case [36]. Indeed, we start from the equations of motion

$$\partial^\mu \partial_\mu A_\nu^a - \left(1 - \frac{1}{\xi}\right) \partial_\nu (\partial^\mu A_\mu^a) + g f^{abc} A^{b\mu} (\partial_\mu A_\nu^c - \partial_\nu A_\mu^c) + g f^{abc} \partial^\mu (A_\mu^b A_\nu^c) + g^2 f^{abc} f^{cde} A^{b\mu} A_\mu^d A_\nu^e = -j_\nu^a. \quad (5)$$

assuming again a current expansion. If we fix the gauge to a Lorenz gauge that is equivalent to the Landau gauge in quantum field theory, we note that the homogeneous equations can be solved by setting $A_\mu^a(x) = \eta_\mu^a \phi(x)$ being η_μ^a a set of constants. Then, the homogeneous equations collapse to $\partial^\mu \partial_\mu \phi + Ng^2 \phi^3 = -j_\phi$ and we have turned back to the previous scalar field theory (this is no more true for other gauges where the correspondence is just an asymptotic one [12] and the exact solutions have not all identical components [36]). In this way, the gluon propagator in the Landau gauge is straightforwardly obtained from eq.(3) setting $\lambda = Ng^2$ and with a factor $\delta_{ab}(\eta_{\mu\nu} - p_\mu p_\nu / p^2)$. These solutions confirm that also Yang-Mills theories seem to share a trivial infrared fixed point. This is supported by lattice studies of the running coupling [37] from lattice at 64^4 and 80^4 with $\beta = 5.7$ where the running coupling is seen to go to zero as momenta lower. A similar result was obtained in [38]. This latter computation shows a perfect consistency with an instanton liquid model in agreement with our scenario.

We now consider quantum field theory. Generating functional for the scalar field can be managed by rescaling the space-time coordinates as $x \rightarrow \sqrt{\lambda}x$ taking a strong coupling expansion $\phi = \sum_{n=0}^{\infty} \lambda^{-n} \phi_n$. At the leading order we need to solve the equation $\square \phi_0 + \lambda \phi_0^3 = j$. Expanding in powers of the current, leading order is just a Gaussian generating functional with the propagator given by eq.(3). Next-to-leading order can be also computed [35]. Turning the attention to Yang-Mills generating functional we realize that it also takes the simple Gaussian form

$$Z_0[j] = N \exp \left[\frac{i}{2} \int d^4x' d^4x'' j^{a\mu}(x') D_{\mu\nu}^{ab}(x' - x'') j^{b\nu}(x'') \right]. \quad (6)$$

using the current expansion $A_\mu^a = \Lambda \int d^4x' D_{\mu\nu}^{ab}(x - x') j^{b\nu}(x') + O(1/\sqrt{N}g) + O(j^3)$ and the

propagator in the Landau gauge $D_{\mu\nu}^{ab}(p) = \delta_{ab} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) \Delta(p)$ being $\Delta(p)$ given by eq.(3). This propagator represents a sum of propagators of a free theory and a mass spectrum of glue excitations identical to that of a harmonic oscillator. Ghost field just decouples in this limit yielding a free massless propagator. All these properties of the quantum Yang-Mills field correspond to the so-called “decoupling solution” [11, 39, 40] (see also [41] for a discussion). This kind of propagator is the one recovered in lattice computations [3–5]. We yield a comparison in Fig.1

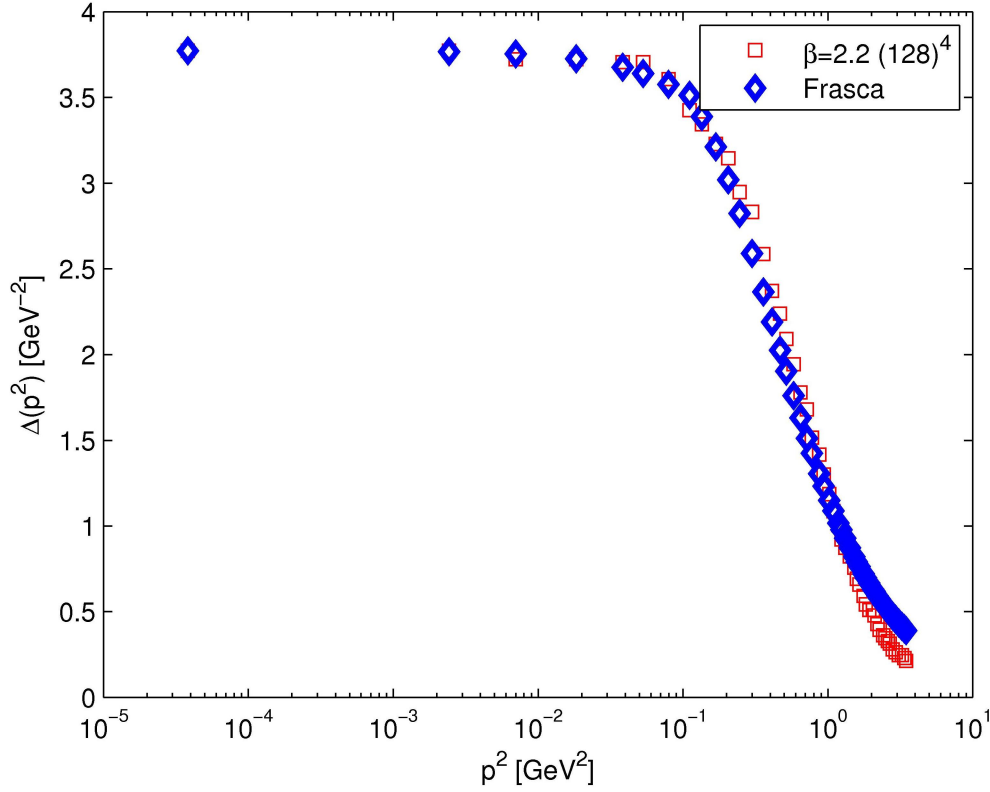


FIG. 1. Comparison of our propagators with the lattice data for SU(2) given in [4] for $(128)^4$ points.

The agreement is exceedingly good as expected.

This results permits to derive the low-energy limit of QCD and this coincides with a nonlocal Nambu-Jona-Lasinio model [14–17]

$$S = \int d^4x \left[\frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_0^2\sigma^2 \right] + S_q \quad (7)$$

where the σ field arises from the gluon propagator in the Gaussian generating functional of the Yang-Mills action, neglecting higher order excited state in the superimposed harmonic

oscillator spectrum being exponential damped. This is the contribution arising from the mass gap of the theory, being $m_0 = (\pi/2K(i))\sqrt{\tilde{\sigma}}$. We are assuming $\tilde{\sigma}$ as the string tension ($\approx (440 \text{ MeV})^2$). Quark fields yield

$$S_q = \sum_q \int d^4x \bar{q}(x) \left[i\not{\partial} - m_q - g \sqrt{\frac{B_0}{3(N_c^2 - 1)}} \eta_\mu^a \gamma^\mu \frac{\lambda^a}{2} \sigma(x) \right] q(x) \quad (8)$$

$$- g^2 \int d^4x' \Delta(x - x') \sum_q \sum_{q'} \bar{q}(x) \frac{\lambda^a}{2} \gamma^\mu \bar{q}'(x') \frac{\lambda^a}{2} \gamma_\mu q'(x') q(x) + O\left(\frac{1}{\sqrt{N}g}\right) + O(j^3).$$

Then, our nonlocal Nambu-Jona-Lasinio model coincides with that presented in [20], directly from QCD, provided the form factor is

$$\mathcal{G}(p) = -\frac{1}{2}g^2\Delta(p) = -\frac{1}{2}g^2 \sum_{n=0}^{\infty} \frac{B_n}{p^2 - (2n+1)^2(\pi/2K(i))^2\tilde{\sigma} + i\epsilon} = \frac{G}{2}\mathcal{C}(p) \quad (9)$$

being B_n obtained from eq.(3), $\mathcal{C}(0) = 1$ and $2\mathcal{G}(0) = G$ the standard Nambu-Jona-Lasinio coupling, fixing in this way the value of G through the gluon propagator. In Fig.2, we compare this form factor both with the one from an instanton liquid [42] that is

$$\mathcal{C}_I(p) = p^2 d^2 \left\{ \pi \frac{d}{d\xi} [I_0(\xi)K_0(\xi) - I_1(\xi)K_1(\xi)] \right\}^2 \quad \text{with } \xi = \frac{|p|d}{2} \quad (10)$$

being I_n and K_n Bessel functions. In the following we normalize this function to be 1 at zero momenta dividing it by $\mathcal{C}_I(0)$.

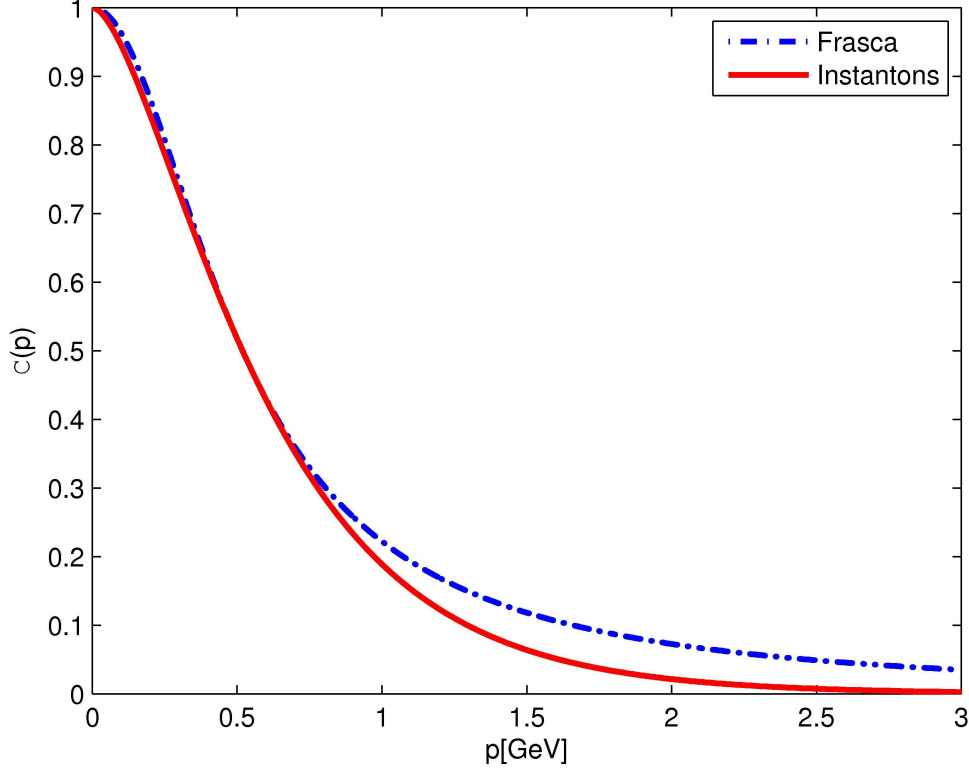


FIG. 2. Comparison of our form factor with that provided in [42] for $\sqrt{\sigma} = 0.417 \text{ GeV}$ and $d^{-1} = 0.58 \text{ GeV}$.

The result is strikingly good for the latter showing how consistently our technique represents Yang-Mills theory through instantons. In the low-energy limit recovers a nonlocal Nambu-Jona-Lasinio model and maintains the defects of this approximation as a non-confining behavior. Higher order corrections can grant to recover this property of the theory. With our approach these can be computed.

So, finally we write down the NJL action we will use in the following as was obtained from QCD

$$\begin{aligned}
S_q = & \sum_q \int d^4x \bar{q}(x) [i\not{\partial} - m_q] q(x) \\
& + \int d^4x \int d^4x' \mathcal{G}(x - x') \sum_q \sum_{q'} \bar{q}(x) \frac{\lambda^a}{2} \gamma^\mu \bar{q}'(x') \frac{\lambda^a}{2} \gamma_\mu q'(x') q(x).
\end{aligned} \tag{11}$$

This can be bosonized in a standard way [43, 44] giving the effective field theory. One introduces the field $\sigma(x) = G\bar{q}(x)q(x)$ and $\boldsymbol{\pi}(x) = G\bar{q}(x)\gamma^5\boldsymbol{\tau}q(x)$ and this will yield, after a

Fierz rearrangement and considering a two flavor QCD,

$$\begin{aligned}
S_{NJL} = & \int d^4x \frac{1}{2}(\partial\sigma)^2 - \frac{1}{2}m_0^2\sigma^2 \\
& + \int d^4x \sum_{q=\{u,d\}} \bar{q}(i\not{\partial} - g(\sigma + i\gamma^5\boldsymbol{\pi} \cdot \boldsymbol{\tau})q) \\
& - \frac{1}{2} \int d^4x \int d^4x' \mathcal{C}(x-x') (\sigma^2(x') + \boldsymbol{\pi}(x') \cdot \boldsymbol{\pi}(x')).
\end{aligned} \tag{12}$$

being $\boldsymbol{\tau}$ SU(2) Pauli matrices and neglecting quark masses. This appears as a well-known quark-meson model and so, we can add the chemical chiral potential as in [21]

$$S_{NJLc} = S_{NJL} + \int d^4x \sum_{q=\{u,d\}} \mu_5 \bar{q} \gamma^0 \gamma^5 q. \tag{13}$$

Finally, we will perform all the computations at finite temperature.

III. CRITICAL TEMPERATURE

The potential has the form [20]

$$\begin{aligned}
V(\sigma, \boldsymbol{\pi}) = & -i\text{Tr} \ln [1 - (i\not{\partial} - \hat{m} - \gamma_0 \gamma_5 \mu_5)^{-1}(\sigma + i\gamma^5 \boldsymbol{\pi} \cdot \boldsymbol{\tau})] \\
& + \int d^4x \left[\frac{1}{2}(G^{-1} + m_0^2)\sigma^2 + \frac{1}{2G} \boldsymbol{\pi} \cdot \boldsymbol{\pi} \right]
\end{aligned} \tag{14}$$

that yields the gap equation [20]

$$M(\boldsymbol{p}s + \mu_5) = \mathcal{C}(|\boldsymbol{p}|s + \mu_5)v. \tag{15}$$

being v the vacuum expectation value of the σ field. We introduce a sum on the Matsubara frequencies $\omega_k = (2k+1)T$ and the gap equation becomes

$$v = \frac{4N_c N_f}{m_0^2 + 1/G} \beta^{-1} \sum_{k=-\infty}^{\infty} \sum_{s=\pm 1} \int \frac{d^3p}{(2\pi)^3} \mathcal{C}(\omega_k, |\boldsymbol{p}|s + \mu_5) \frac{M(\omega_k, |\boldsymbol{p}|s + \mu_5)}{\omega_k^2 + (|\boldsymbol{p}|s + \mu_5)^2 + M^2(\omega_k, |\boldsymbol{p}|s + \mu_5)}. \tag{16}$$

Here $\mathcal{C}(p)$ is given by $\frac{G}{2}\mathcal{C}(p) = \mathcal{G}(p)$ using eq.(9) but moving to Euclidean. The restoration of chiral symmetry is given at $v = 0$ and so, we have to solve

$$1 = \frac{4N_c N_f}{m_0^2 + 1/G} \beta^{-1} \frac{g^4}{G^2} \sum_{k=-\infty}^{\infty} \sum_{s=\pm 1} \int \frac{d^3p}{(2\pi)^3} \mathcal{C}^2(\omega_k, |\boldsymbol{p}|s + \mu_5) \frac{1}{\omega_k^2 + (|\boldsymbol{p}|s + \mu_5)^2} \tag{17}$$

to obtain the critical temperature as a function of μ_5 . We consider just one term in the form factor (9). This is so because we want to be consistent with the NJL action just obtained, noting that higher excitations are exponentially damped. Then, we will have

$$\mathcal{C}(p) = \frac{g^2}{G} \frac{B_0}{p^2 + m_0^2} \quad (18)$$

having moved to Euclidean and being $m_0 = (\pi/2K(i))\sqrt{\bar{\sigma}}$ the mass gap. We take $Z = g^2 B_0/G$ and then

$$1 = \frac{4N_c N_f}{m_0^2 + 1/G} \beta^{-1} \sum_{k=-\infty}^{\infty} \sum_{s=\pm 1} \int \frac{d^3 p}{(2\pi)^3} \frac{Z^2}{(\omega_k^2 + (|\mathbf{p}|s + \mu_5)^2 + m_0^2)^2} \frac{1}{\omega_k^2 + (|\mathbf{p}|s + \mu_5)^2}. \quad (19)$$

Matsubara sum can be performed analytically giving

$$\begin{aligned} \mathcal{I}_{p,s} = & \beta \frac{\pi}{2m_0^4 ||\mathbf{p}|s + \mu_5|} \tanh\left(\frac{\pi}{2}\beta ||\mathbf{p}|s + \mu_5|\right) \\ & + \beta^2 \frac{\pi^2}{8((|\mathbf{p}|s + \mu_5)^2 + m_0^2)m_0^2} \\ & - \beta^2 \frac{\pi^2}{8((|\mathbf{p}|s + \mu_5)^2 + m_0^2)m_0^2} \tanh^2\left(\frac{\pi}{2}\beta \sqrt{(|\mathbf{p}|s + \mu_5)^2 + m_0^2}\right) \\ & - \beta \frac{\pi}{4} \frac{2(|\mathbf{p}|s + \mu_5)^2 + 3m_0^2}{((|\mathbf{p}|s + \mu_5)^2 + m_0^2)^{\frac{3}{2}}m_0^4} \tanh\left(\frac{\pi}{2}\beta \sqrt{(|\mathbf{p}|s + \mu_5)^2 + m_0^2}\right). \end{aligned} \quad (20)$$

In order to get an understanding, we try to solve the gap equation with $\mu_5 = 0$ and then, we restate it into the equation. We will have

$$\begin{aligned} \mathcal{I}_{p,1} = & \beta \frac{\pi}{2m_0^4 p} \tanh\left(\frac{\pi}{2}\beta p\right) \\ & + \beta^2 \frac{\pi^2}{8(p^2 + m_0^2)m_0^2} \\ & - \beta^2 \frac{\pi^2}{8(p^2 + m_0^2)m_0^2} \tanh^2\left(\frac{\pi}{2}\beta \sqrt{p^2 + m_0^2}\right) \\ & - \beta \frac{\pi}{4} \frac{2p^2 + 3m_0^2}{(p^2 + m_0^2)^{\frac{3}{2}}m_0^4} \tanh\left(\frac{\pi}{2}\beta \sqrt{p^2 + m_0^2}\right) \end{aligned} \quad (21)$$

that yields for $\beta \rightarrow 0$, after integration on momenta,

$$\beta^2 \frac{\pi^2}{8m_0^2} \Lambda \quad (22)$$

being Λ a needed cut-off to regularize divergent integrals. This cut-off must be chosen so that the product $\beta\Lambda$ is kept constant while Λ runs to infinity. Then,

$$T_c = Z^2 \frac{N_c N_f}{m_0^2 + 1/G} \frac{g^4}{G^2} \frac{\pi^2}{2m_0^2} \Lambda. \quad (23)$$

We see that, in this case, the gap equation admits always a solution whatever is the coupling g and chiral symmetry is broken. As expected, temperature runs to infinity as the cut-off itself.

When μ_5 is turned on, one has, taken limit $\beta \rightarrow 0$,

$$\begin{aligned} \mathcal{I}_{p,s} = & \beta^2 \frac{\pi^2}{4m_0^4} \\ & + \beta^2 \frac{\pi^2}{8((|\mathbf{p}|s + \mu_5)^2 + m_0^2)m_0^2} \\ & - \beta^2 \frac{\pi^2}{8} \frac{2(|\mathbf{p}|s + \mu_5)^2 + 3m_0^2}{((|\mathbf{p}|s + \mu_5)^2 + m_0^2)m_0^4}. \end{aligned} \quad (24)$$

and, after integration on momenta, we get

$$K(\beta, \mu_5) = \beta^2 \frac{1}{4m_0^2} \Lambda + \beta^2 \frac{\pi}{8m_0^3} (\mu_5^2 - m_0^2). \quad (25)$$

This yields

$$T_c = Z^2 \frac{N_c N_f}{m_0^2 + 1/G} \frac{g^4}{G^2} \left(\frac{1}{4m_0^2} \Lambda + \frac{\pi}{8m_0^3} (\mu_5^2 - m_0^2) \right). \quad (26)$$

This is the main result of the paper showing that the critical temperature increases with the chiral chemical potential in agreement with Ruggieri and Peng [32], with lattice results [27, 28] and solution of Dyson-Schwinger equations [31].

It is interesting to note the dependence on the mass gap m_0 . This equation seems to imply that $|\mu_5| \geq m_0$.

IV. CONCLUSIONS

Using recent studies on lattice, we were able to derive the low-energy limit of QCD. The result is given by a nonlocal NJL model that is amenable to analytical computations. In this way, we are able to conclude that the critical temperature for chiral symmetry breaking in QCD increases as the square of the chiral chemical potential in agreement with recent lattice studies. This result supports the conclusions presented in a recent work [32] supporting a preferential choice of a renormalization scheme.

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